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Naturally heavy superpartners and a Little Higgs

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Abstract: We construct an extension of the MSSM in which scalar superpartners can naturally be as heavy as 1 TeV. In the MSSM, the most significant fine tuning stems from the logarithmically enhanced top-stop loop contribution to the soft Higgs mass. We combine supersymmetry with the "simplest little Higgs" to render this loop finite, thereby removing the large logarithm even in models in which superpartner masses are generated at high scales such as in supergravity. Our model predicts an extended Higgs sector, superpartner masses near a TeV and little Higgs partners at a few TeV.

Keywords: Higgs Physics, Beyond Standard Model, Supersymmetric Standard Model.

Contents

1. Introduction

In the Standard Model (SM) the Higgs mass receives quadratically divergent quantum contributions. This indicates that the SM is only a low-energy description of a more complete UV theory in which the divergences are canceled by new particles. To avoid fine tuning the masses of the new particles Λ must be low enough to cancel the quadratic divergence before it becomes significantly bigger than the electroweak scale. For particles which couple with coupling constants of order 1 to the Higgs this implies the bound

$$\Lambda^2 \frac{1}{16\pi^2} \lesssim m_{\rm Higgs}^2 \ . \qquad (1.1)$$

Numerically, one finds $\Lambda \lesssim$ 2-5 TeV, within reach of precision electroweak measurements. No significant deviations from the SM have been seen which implies constraints on the couplings of these heavy particles.

In supersymmetry (SUSY), the new particles which cancel quadratic divergences are the superpartners. An important feature of the minimal supersymmetric standard model (MSSM) is that it is renormalizable. This means that no new physics is required until extremely high scales, such as M_{GUT} or M_{Planck}. This is nice because it allows the MSSM to avoid problems with flavor changing neutral currents (FCNCs) if we make the further assumption of flavor universal soft SUSY breaking. A problem with the MSSM however is that renormalization from all scales above the superpartner masses (M_{SUSY}) enhances the one-loop contributions to the Higgs mass parameter by a large logarithm and eq. (1.1) is replaced by

$$M_{SUSY}^2 \frac{1}{16\pi^2} \, \mathrm{Log}\left(\frac{M_{\text{Planck}}^2}{M_{SUSY}^2}\right) \lesssim m_{\text{Higgs}}^2 \, . \tag{1.2}$$

The large logarithm almost cancels the $1/16\pi^2$ and therefore naturalness requires superpartner masses of order m_{Higgs}. Naively, this is a disaster for the MSSM because it predicts a large number of superpartners at the weak scale but none have been observed. The problem is greatly alleviated by R-parity [1]. This symmetry implies that superpartners can only by produced in pairs, and it eliminates all tree level contributions from SUSY to precision electroweak observables. Nonetheless, the absence of any evidence for superpartners is putting severe constraints on parameter space.

Another phenomenological problem of the MSSM is that in most of parameter space the quartic Higgs coupling is too small and a physical Higgs mass below the experimental bound is predicted. In the MSSM the only way out of this problem is to increase the stop mass. This raises the Higgs mass because the Higgs quartic receives contributions from a top-stop loop proportional to $Log(m_{stop}/m_{\text{top}})$, and it also allows larger corrections from A-terms. A sufficiently large physical Higgs mass can be obtained with stop masses larger than $1/2\,\text{TeV}$, but this comes at the cost of significant fine tuning because of large contributions of the stops to eq. (1.2). Various recent attempts to solve the fine tuning problem in the MSSM may be found in [2 − 7] and references therein.

The aim of this paper is to construct a supersymmetric model in which superpartners can be heavy without destroying naturalness. The new ingredient is to introduce a global symmetry into the Higgs sector of the MSSM which is spontaneously broken at a few TeV. The lightest Higgs doublet of the model arises as a pseudo-Nambu-Goldstone boson (pNGB) in this symmetry breaking. Examples of these so-called little-Higgs model building in non-supersymmetric case may be found in [8 − 20]. The combination of supersymmetry and the global symmetry removes the logarithmic divergence in the soft Higgs mass eq. (1.2). This allows superpartner masses to be raised to near $4\pi M_{weak}$ without significant tuning and explains why no superpartners have been observed. It also increases the loop contributions to the Higgs mass from the heavy stops.

We construct our model by supersymmetrizing the "simple group" little Higgs [11 − 13]. The little Higgs mechanism and SUSY combine to naturally generate a 4π hierarchy between the weak scale and the "partner" scale (little Higgs partners and superpartners). Unfortunately, this model with an $\text{SU}(3)_{weak} \times \text{U}(1)$ gauge group and a minimal Higgs content predicts $\tan\beta = 1$ which implies two problems: the usual tree level quartic Higgs coupling vanishes and the predicted top Yukawa coupling is too small. Our full model has a tree level quartic coupling for the Higgs which comes from a supersymmetric version of the "missing vev" potential [11, 21], and we increase the top Yukawa couplings infrared quasi-fixed point by splitting color SU(3) into SU(3) × SU(3) at high energies.

At the weak scale, our model reduces to the Standard Model with a relatively light Higgs ($m_h \lesssim 200\,\text{GeV}$). Superpartners and an extended Higgs sector are near a TeV while little Higgs partners are in the few TeV range. This places the new physics out of reach of

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current accelerators but predicts a rich physics program for the LHC. In addition to the well-known SUSY signatures, the LHC might see signs of the little Higgs quark partners (T', S', D') and the $\mathrm{SU}(3)_{weak}$ gauge boson partners (W' and Z') which have masses in the range of 1-5 TeV.

The supersymmetrized "simple group" little Higgs model is presented in section 2. Section 3 contains the fully realistic model with a tree level quartic and section 4 contains the phenomenology and our conclusion.

2. The simplest little SUSY

The goal of this work is to construct a supersymmetric alternative to the MSSM where the superpartners are parametrically heavier than the electroweak symmetry breaking scale by a factor of 4π without fine tuning of the Higgs mass. In our model fine tuning is reduced relative to the MSSM, and the model remains perturbative up to the Planck scale.

Our approach will be to construct a supersymmetric generalization of a little Higgs theory where the quadratic divergence in the Higgs mass is canceled because of a collectively broken approximate global symmetry. A new difficulty arises because of our goal of constructing a UV complete theory which remains well-defined all the way to M_{Planck}. The problem stems from renormalization group running between M_{Planck} and M_{weak}. Collectively broken symmetries imposed at the Planck scale may in fact be completely broken at low energies because perturbatively small loop effects get enhanced by large logs. Consider for example, an exact global symmetry $\mathbf{G}$ imposed at M_{Planck}, but with a subgroup $\mathbf{H}$ weakly gauged. As we renormalize the theory down to low energies, wave function renormalization due to $\mathbf{H}$ interactions breaks the $\mathbf{G}$ symmetry and — because of the large $\log(M_{\mathrm{Planck}}/M_{weak})$ — only $\mathbf{H}$ remains as a symmetry near the weak scale. This problem appears to be generic to models in which a subgroup of a simple global symmetry is gauged (e.g. $[\mathrm{SU}(2)]^2 \subset \mathrm{SU}(5)$ in the Littlest Higgs or $[\mathrm{SU}(2)]^2 \subset \mathrm{SU}(6)$ in the SU(6)/SP(6) little Higgs). A naturally light Higgs mass in this framework can only occur if the soft masses are themselves small enough. An alternative solution is to rely on a susy-breaking mechanism which generates soft masses at low energy (gauge mediation as an example). In these models naturalness constraints the scale of susy breaking to be around 100 TeV [4, 22].

We therefore base our model on the Simplest Little Higgs [12] where the global group $[\mathrm{SU}(3)]^2$ is broken collectively by gauging only the diagonal SU(3). The nice feature here is that the log-divergent wave function renormalization due to the SU(3) gauge interactions automatically preserves both SU(3)s.

The object of this section is to construct a concrete model where the above idea is realized. We first construct the Higgs sector and show that it contains a light electroweak doublet which we identify with the SM Higgs doublet. We then show that it has all the necessary interactions with matter as well as self couplings in order to break electroweak symmetry. We show that radiative corrections to the Higgs soft mass remain under control even when superpartner masses are taken large. However, the model predicts $\tan\beta \simeq 1$ which implies that the top Yukawa coupling is too small. Another consequence is that the Higgs quartic coupling vanishes at tree level so that even the large radiative correction due to the top-stop loop is not sufficient to produce a Higgs mass above the current experimental limit.

2.1 Higgs sector and symmetries

As in the Simplest Little Higgs we start with the gauge group $(\mathrm{SU}(3)_c, \mathrm{SU}(3)_w)_{U(1)_X}$ in the UV which is broken to the MSSM gauge group at scale f. Third generation quarks and all leptons are embedded in the following $(\mathrm{SU}(3)_c, \mathrm{SU}(3)_w)_{U(1)_X}$ representations

$$\begin{aligned} \Psi_{Q_3} &= (3,\bar{3})_{\frac{1}{3}} & \Psi_L &= (1,\bar{3})_{-\frac{1}{3}} \\ B^c &= (\bar{3},1)_{\frac{1}{3}} & E^c &= (1,1)_1 \\ T^c_{1,2} &= (\bar{3},1)_{-\frac{2}{3}} & N^c &= (1,1)_0 \ . \end{aligned} \tag{2.1}$$

The MSSM Higgs fields are embedded in fields Φ and their adjoints $\bar{\Phi}$ with quantum numbers

$$\Phi_1, \Phi_2 = (1,3)_{\frac{1}{3}} \qquad \text{and} \qquad \bar{\Phi}_1, \bar{\Phi}_2 = (1,\bar{3})_{-\frac{1}{3}} \ . \tag{2.2}$$

Keeping in mind the symmetry breaking pattern $\mathrm{SU}(3)_w \times U(1)_X \to \mathrm{SU}(2)_w \times U(1)_Y$ we designate the components of Φ as

$$\Phi_i \equiv \begin{pmatrix} H_i \\ S_i \end{pmatrix}; \qquad \bar{\Phi}_i \equiv \begin{pmatrix} \bar{H}_i & \bar{S}_i \end{pmatrix} \ . \tag{2.3}$$

Here H_i and $\bar{H}_i$ are $\mathrm{SU}(2)_w$ doublets with hypercharge $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. $\langle S_i \rangle$ and $\langle \bar{S}_i \rangle$ break the gauge group $\mathrm{SU}(3)_w \times U(1)_X$ down to $\mathrm{SU}(2)_w \times U(1)_Y$. The simplest superpotential which forces this symmetry breaking is

$$W_{\text{Higgs}} = X_1(\bar{\Phi}_1 \Phi_1 - f_1^2) + X_2(\bar{\Phi}_2 \Phi_2 - f_2^2), \tag{2.4}$$

where X_1 and X_2 are two gauge singlets. Supersymmetry is broken softly as in the MSSM. For reasons which have to do with the D-term potential and which are discussed at the end of this subsection we assume that the scalar components ϕ and $\bar{\phi}$ of the superfields Φ and $\bar{\Phi}$ have identical soft masses

$$V_{soft} = \tilde{M}_1^2(|\phi_1|^2 + |\bar{\phi}_1|^2) + \tilde{M}_2^2(|\phi_2|^2 + |\bar{\phi}_2|^2) \ . \tag{2.5}$$

We will justify this assumption with an approximate Z_2 symmetry between Φ and $\bar{\Phi}$ which may be the remnant of a larger unifying symmetry at higher scales.

For simplicity, throughout this section we work in the limit where the soft masses are much smaller than the $\mathrm{SU}(3)_w$ breaking scale f. The important result that the SM Higgs field is a pseudo-Nambu-Goldstone boson (pNGB) is independent of this limit. However, taking soft masses much smaller than f has the advantage that the Φ_i fields may be expanded around the supersymmetry preserving expectation values f_i, thereby keeping intermediate formulas manifestly supersymmetric

$$\langle \phi_1 \rangle = \langle \bar{\phi}_1^\dagger \rangle = \begin{pmatrix} 0 \\ f_1 \end{pmatrix}, \qquad \langle \phi_2 \rangle = \langle \bar{\phi}_2^\dagger \rangle = \begin{pmatrix} 0 \\ f_2 \end{pmatrix} \ . \tag{2.6}$$

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We find that two linear combinations of doublets in the Φ and $\bar{\Phi}$s are eaten by the super-Higgs mechanism

$$\begin{aligned} H_{eat} &\equiv \frac{1}{f}\Big[f_1 H_1 + f_2 H_2\Big] \\ \bar{H}_{eat} &\equiv \frac{1}{f}\Big[f_1 \bar{H}_1 + f_2 \bar{H}_2\Big] \ , \end{aligned} \tag{2.7}$$

where $f \equiv \sqrt{f_1^2 + f_2^2}$. The two orthogonal combinations remain massless in the supersymmetric limit

$$\begin{aligned} H_u &\equiv \frac{1}{f}\Big[f_2 H_1 - f_1 H_2\Big] \\ H_d &\equiv \frac{1}{f}\Big[f_2 \bar{H}_1 - f_1 \bar{H}_2\Big] \ . \end{aligned} \tag{2.8}$$

We now turn on soft SUSY breaking and expand the potential including eq. (2.4), eq. (2.5), and the D-term around the minima in eq. (2.6). After integrating out all scalars with masses of order f we arrive at an effective low energy potential for the light doublets

$$V_{soft}^{\rm eff} \approx (\tilde{M}_1^2 + \tilde{M}_2^2)\ |H_u - H_d^\dagger|^2 = 2(\tilde{M}_1^2 + \tilde{M}_2^2)\,|\tilde{H}|^2\ , \tag{2.9}$$

where

$$H \equiv \frac{1}{\sqrt{2}}\Big[H_u + H_d^\dagger\Big] \tag{2.10}$$

$$\tilde{H} \equiv \frac{1}{\sqrt{2}}\Big[H_u - H_d^\dagger\Big]\ . \tag{2.11}$$

The doublet H remains massless, it is protected by the little Higgs mechanism. $\tilde{H}$ is very heavy and does not contribute significantly to electroweak symmetry breaking. The light field H corresponds to the SM Higgs. Note that a vacuum expectation value for H implies equal vevs for H_u and H_d, i.e. $\tan\beta = 1$.

For the remainder of this subsection we give an alternative derivation of the masslessness of the little Higgs. Consider a similar theory in which two SU(3) symmetries are gauged, one acting on $\Phi_1, \bar{\Phi}_1$ and the other acting on $\Phi_2, \bar{\Phi}_2$. The two SU(3) sectors are completely decoupled and can be analyzed separately. Both have a single exact SU(3) symmetry which is spontaneously broken, leading to a set of NGBs which are eaten by the Higgs mechanism. Note that SUSY is broken by the soft masses and we expect no further massless scalar doublets.

Now recall our assumption of identical soft masses for ϕ_i and $\bar{\phi}_i$. This implies identical vevs and therefore a vanishing expectation value for the D-term (note, $D^a = \phi_1^\dagger t^a \phi_1 - \bar{\phi}_1 t^a \bar{\phi}_1^\dagger + 1 \leftrightarrow 2$). and only one linear combination of doublets obtains a mass from the D-term potential $(D^a)^2$. This combination is $H_{eat} - \bar{H}_{eat}^\dagger$. Note that this would not be true if D^8 and D^x had supersymmetry breaking expectation values, then our little Higgs would also get a mass.

Finally, let us return to the theory with a single gauged SU(3). The scalar potential of this theory is identical except that there is now a single D-term containing both sets of fields. But as before, at quadratic order, this D-term does not depend on the linear combinations $H_1 + \bar{H}_1^\dagger$ and $H_2 + \bar{H}_2^\dagger$. The sum of these two correspond to the eaten doublet, the difference remains massless, it is the little Higgs doublet H. This argument can be extended to show that the little Higgs also does not obtain a quartic self-coupling from any terms in the tree-level potential either.

2.2 Yukawa couplings

Yukawa couplings involving the top quark are obtained from the superpotential

$$W_{\text{top}} = Y_1 \Psi_{Q_3} \Phi_1 T_1^c + Y_2 \Psi_{Q_3} \Phi_2 T_2^c \ . \tag{2.12}$$

where Y_i are coupling constants of order 1. After $\mathrm{SU}(3)_w \times U(1)_X \to \mathrm{SU}(2)_w \times U(1)_Y$ breaking, the triplet Ψ_{Q_3} is reducible. We denote the irreducible pieces as

$$\Psi_{Q_3} \equiv \Big(Q_3 \ T \Big) . \tag{2.13}$$

Eq. (2.12) gives mass to T

$$W_{\text{top}} \supset T(Y_1 f_1 T_1^c + Y_2 f_2 T_2^c) \ . \tag{2.14}$$

The MSSM quark singlet T^c is identified with the orthogonal combination of T_1^c and T_2^c which remains massless

$$T^c \equiv \frac{1}{\sqrt{Y_1^2 f_1^2 + Y_2^2 f_2^2}} \Big(Y_2 f_2 T_1^c - Y_1 f_1 T_2^c \Big) . \tag{2.15}$$

The SM top quark Yukawa coupling is obtained from eq. (2.12) by expanding to first order in the Higgs field

$$y_t = \frac{Y_1 Y_2 f}{\sqrt{2}\sqrt{Y_1^2 f_1^2 + Y_2^2 f_2^2}} \ . \tag{2.16}$$

Note that $U(1)_X$ and $\mathrm{SU}(3)_w$ quantum numbers forbid a renormalizable Yukawa coupling for the bottom quark. We therefore introduce an additional triplet $\chi = (1,3)_{-\frac{2}{3}}$ and its adjoint $\bar{\chi} = (1,\bar{3})_{\frac{2}{3}}$ and write

$$W_\chi = -M\chi\bar{\chi} + Y_b \Psi_{Q_3} \chi B^c + \eta_1 \chi \Phi_1 \Phi_2 + \eta_2 \bar{\chi} \bar{\Phi}_1 \bar{\Phi}_2 \ . \tag{2.17}$$

Here the $\mathrm{SU}(3)_w$ indices in $\chi\Phi_1\Phi_2$ and in $\bar{\chi}\bar{\Phi}_1\bar{\Phi}_2$ are contracted anti-symmetrically with an epsilon tensor. After integrating out χ at the scale $M \gg f$, we obtain a Yukawa coupling for the bottom quark as well as a μ-term for the Higgses

$$\begin{aligned} W_\chi^{\text{eff}} &= \frac{Y_b \eta_2}{M} \Psi_{Q_3} (\bar{\Phi}_1 \bar{\Phi}_2) B^c + \frac{\eta_1 \eta_2}{M} (\Phi_1 \Phi_2)(\bar{\Phi}_1 \bar{\Phi}_2) \\ &\supset \frac{Y_b \eta_2 f}{M} Q_3 H_d B^c + \frac{\eta_1 \eta_2 f^2}{M} H_d H_u \ . \end{aligned} \tag{2.18}$$

Note that since we used antisymmetric contractions, this part of the potential does not change the vevs of $\mathcal{O}(f)$. As in the MSSM, the μ-term contributes to the Higgs potential and gives masses to the Higgsinos.

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2.3 The Higgs mass and naturalness

The standard model Higgs mass is related to its quartic coupling λ by $m_h = \sqrt{\lambda} v$ where v is the Higgs vev. Thus to determine the Higgs mass, we need to know the quartic coupling. In our theory, the Higgs is a pNGB and in the exact symmetry limit, the quartic vanishes.

A contribution to the quartic comes from the $(D^a)^2$-potential. When the D-term vanishes this potential is insensitive to supersymmetry breaking, and after integrating out the states with masses of order f, the remaining potential is simply the MSSM D-term

$$\sum_a (H_u^\dagger t^a H_u - H_d t^a H_d^\dagger)^2 = \sum_a (H^\dagger t^a \tilde{H} + \tilde{H}^\dagger t^a H)^2 \ , \tag{2.19}$$

where t^a are the SU(2) × U(1) generators. This looks good, however there is a problem. Recall from the previous subsection that $\tilde{H}$ has a mass of order the soft SUSY masses $\tilde{M}$ which are large. Therefore the vev for $\tilde{H}$ vanishes to lowest order and the D-term potential is flat in the H direction. This should be familiar from the MSSM where the tree level quartic also vanishes for $\tan\beta = 1$.

Below the superpartner masses, the Higgs quartic gets its usual loop contributions [23–25]. The most important comes from the top quark and is cut off at the stop mass

$$\delta\lambda \simeq \frac{3y_t^4}{8\pi^2} \ln \frac{\tilde{m}_t^2}{m_t^2} \ . \tag{2.20}$$

We see that the Higgs mass rises logarithmically with the stop mass, so that there should be a critical stop mass for which this contribution is large enough to lift the Higgs mass above the current bound $m_h > 114\,\text{GeV}$ [26]. Unfortunately, the rise is very slow [27] and even stop masses as large as 1 TeV are not sufficient. In the next section we add new fields and interactions to the Higgs sector which give a tree level contribution to the quartic coupling. We will see that Higgs masses between the experimental bound and about 200 GeV are expected.

A closely related top-stop loop diagram gives a negative contribution to the soft mass parameter of the Higgs and can trigger electroweak symmetry breaking

$$\delta m^2 \simeq -\frac{3y_t^2}{8\pi^2} \tilde{m}_t^2 \ln \frac{f^2}{\tilde{m}_t^2} \ . \tag{2.21}$$

Note that unlike in the MSSM the logarithm in this expression is cut off at the SU(3) breaking scale f, i.e. the logarithm is much smaller than in the MSSM where we would have $\ln(M_{\text{Planck}}^2/\tilde{m}_t^2)$ instead. This is the main accomplishment of the little Higgs mechanism in this context. The little Higgs partners render the stop loop finite above the scale f and therefore the Higgs mass is insensitive to the large scale M_{Planck}. Plugging in numbers we see that stop masses as large as 1 TeV do not lead to significant fine-tuning.

3. Extending simplest little SUSY

The accomplishment of our model presented in the previous section is to render the Higgs soft mass UV insensitive, that is to remove the large logarithm which multiplies the Higgs

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soft mass in the MSSM and leads to excessive fine tuning. The model also allows superpartner masses to be parametrically larger than the electroweak scale, thus explaining why we have not seen any signs of supersymmetry at colliders. In this section we aim to fix the three technical problems which arose in the process.

- This model has a vanishing quartic Higgs coupling at tree level, requiring the Higgs mass to come from the top-stop loop. However, even for stop masses as large as 1 TeV, the Higgs mass appears to be too small. We therefore propose an extension of the simplest model with additional fields Φ_3 and $\bar{\Phi}_3$ which allow for the generation of a tree level quartic.
- Our little Higgs mechanism crucially depends on at least an approximate Z_2 charge conjugation in the Higgs sector. Unequal soft masses for Φ and $\bar{\Phi}$ leads to a D-term of order the soft susy breaking and therefore also a Higgs soft mass of order $\tilde{m}^2$. A charge conjugation symmetry under which $\Phi \leftrightarrow \bar{\Phi}$ is a way out of the problem. However, renormalization due to the top Yukawa coupling strongly breaks this symmetry and — in running from the Planck scale — a large difference of order 1 is generated. To solve this problem we also make the top Yukawa coupling charge conjugation symmetric.
- Another problem is that the top Yukawa couplings Y_1, Y_2 in eq. (2.12) must be very large to ensure a sufficiently large top mass. When running them up into the UV they blow up at around 10^7 GeV. We show that an enlarged color sector allows these couplings to stay perturbative all the way to the Planck scale.

3.1 A tree level quartic

In the previous section we pointed out that except for the small couplings proportional to $\eta_1\eta_2$ in eq. (2.18) the superpotential preserves the two global SU(3) symmetries associated with Φ_1 and Φ_2. As a result, the SM Higgs doublet is a pNGB and receives neither a large mass nor a quartic. W_χ gives a weak scale mass and a tiny quartic. Increasing the η's in order to increase the quartic also raises the mass term, thereby increasing fine tuning. In order to achieve natural electroweak symmetry breaking with a heavy enough physical Higgs we need to find a different source for a tree level quartic coupling.

Any single operator which gives a quartic by violating the two SU(3)s also contributes to the soft mass term. To avoid this we employ the little Higgs trick, collective symmetry breaking. We accomplish this by introducing an extra field Φ_3 which couples to Φ_1 and Φ_2 in two different terms of the superpotential and hence breaks the two SU(3)s collectively. The new superpotential in the Higgs sector is

$$\begin{aligned} W_{\text{Higgs}} = {} & X_1(\bar{\Phi}_1\Phi_1 - f_1^2) + X_2(\bar{\Phi}_2\Phi_2 - f_2^2) + X_3\bar{\Phi}_3\Phi_3 + \\ & + aX_4\bar{\Phi}_1\Phi_3 + bX_5\bar{\Phi}_2\Phi_3 + cX_6\bar{\Phi}_3\Phi_1 + dX_7\bar{\Phi}_3\Phi_2 + W_s \ . \end{aligned} \tag{3.1}$$

W_s is the part of the superpotential which involves only singlets. The choice is not unique. We use the following simple form

$$W_s = \frac{1}{3}X_3^3 - \frac{1}{2}m_3X_3^2 + \frac{1}{3}X_4^3 - \frac{1}{2}m_4X_4^2 + \delta X_5X_6X_7 \ . \tag{3.2}$$

When all the vevs are turned on, the light Higgs doublet is rotated into $\bar{\Phi}_3$ and attains a quartic proportional to c and d. However it does not receive a large soft mass from this mixing as $\tilde{M}_3^2$ - the soft mass of $\bar{\Phi}_3$ - can be much smaller than other soft masses. This is because Φ_3 and $\bar{\Phi}_3$ do not couple to colored particles and therefore the loop corrections to $\tilde{M}_3^2$ are small.

It is most transparent to analyze this theory when $f^2 \gg m_3^2, m_4^2$, and when all supersymmetric masses are larger than soft masses because in this limit we can integrate out all the states associated with the little Higgs mechanism in a supersymmetric fashion. In doing this, we find the NMSSM [28, 29] with a distinctive prediction for superpartner masses at low energies. However our results for the Higgs potential calculated in this section do not depend on taking this limit, the model also works when some of the little Higgs states are lighter than superpartners.

To begin, let us disregard W_s. Then SU(3)$_w$ is broken down to SU(2)$_w$ at the scale f. One doublet is eaten by the heavy gauge fields and one gets mass from the D-term. We end up with 4 light doublets and a several singlets. Now as we turn on W_s, the singlets X_3, X_4 and X_5 attain a non-zero vev. The superpotential in eqs. (3.1, 3.2) gives mass to all but two doublets and singlets and we obtain a variant of the NMSSM at low energies.

$$W_h = \rho S H_d H_u + \delta' S' S^2 \ . \tag{3.3}$$

where we have defined

$$\begin{aligned} H_u &\equiv \frac{1}{f}\left[f_2 H_1 - f_1 H_2\right] \\ H_d &\equiv \frac{1}{f\sqrt{2\mu_4^2+\mu_3^2}}\left[\mu_3(f_2\bar{H}_1 - f_1\bar{H}_2) - \sqrt{2}\mu_4 f \bar{H}_3\right] \ , \end{aligned} \tag{3.4}$$

and also parametrized

$$\begin{aligned} \mu_3 &= \mu_3\langle X_3\rangle, \qquad \sqrt{2}\mu_4 = a\langle X_4\rangle f/f_2 = -b\langle X_5\rangle f/f_1, \\ \rho &= \frac{cdf}{\sqrt{c^2 f_1^2 + d^2 f_2^2}} \frac{\sqrt{2}\mu_4}{\sqrt{2\mu_4^2+\mu_3^2}} \ . \end{aligned} \tag{3.5}$$

Note that in this basis the soft terms are

$$V_{soft} = (\tilde{M}_1^2 + \tilde{M}_2^2)\left|H_u - \frac{\mu_3}{\sqrt{2\mu_4^2+\mu_3^2}} H_d^\dagger\right|^2 + \tilde{M}_3^2 \frac{2\mu_4^2}{2\mu_4^2+\mu_3^2}|H_d|^2 \ . \tag{3.6}$$

Since $\tilde{M}_1^2, \tilde{M}_2^2 \gg \tilde{M}_3^2$, we identify the SM Higgs doublet as

$$H \equiv \frac{\mu_3}{\sqrt{2(\mu_4^2+\mu_3^2)}} H_u + \frac{\sqrt{2\mu_4^2+\mu_3^2}}{\sqrt{2(\mu_4^2+\mu_3^2)}} H_d^\dagger \ . \tag{3.7}$$

Defining the mixing angle $\theta_{34} \equiv \tan^{-1}(\mu_3/\mu_4)$ and using the Higgsino mass $\mu \equiv \eta_1\eta_2 \frac{f^2\mu_3}{M\sqrt{2\mu_4^2+\mu_3^2}}$ we can write the Higgs soft mass and quartic as

$$\begin{aligned} -m^2 &= |\mu|^2 + \tilde{M}_3^2 \cos^2\theta_{34} \ , \\ \lambda &= \left(\frac{|c|^2|d|^2 f^2}{8(|c|^2 f_1^2 + |d|^2 f_2^2)}\right)\sin^2 2\theta_{34} \ . \end{aligned} \tag{3.8}$$

3.2 The Z_2 symmetry and anomalies

At this point, we return to our assumption of equal soft masses for the scalars ϕ and $\bar{\phi}$. We justified this assumption by showing that the Higgs sector has a charge conjugation symmetry under which $\Phi \leftrightarrow \bar{\Phi}$. However, the top Yukawa coupling breaks this symmetry and in running down from the Planck scale a large asymmetry in the soft masses is generated. To solve this problem we modify the top Yukawa coupling to make it charge conjugation symmetric.

We add the multiplets $\Psi_{Q_4} = (3,3)_0$ and $\bar{\Psi}_{Q_4} = (\bar{3},\bar{3})_0$ as well as B_2^c and $\bar{B}_2^c$, where B_2^c has the same quantum numbers as the MSSM B^c. These vector-like multiplets are assumed to have masses near the scale f so that the Z_2 symmetry of the Higgs-top sector is restored above f. The charge conjugate of the top Yukawa coupling eq. (2.12) is then

$$W_{Z_2} = Y_1 \bar{\Phi}_1 \Psi_{Q_4} B^c + Y_2 \bar{\Phi}_2 \Psi_{Q_4} B_2^c \tag{3.9}$$

Of course, the Z_2 is violated by the masses for Ψ_{Q_4} and B_2^c, but the $\Phi - \bar{\Phi}$ soft mass splitting due to this symmetry breaking is small (loop suppressed and not log enhanced).

In extending the model to include first and second generation quarks we must cancel the $\mathrm{SU}(3)_w$ anomalies. This is most easily accomplished by embedding them as [30 − 32, 12]

$$\begin{aligned} \Psi_Q^{(1,2)} &= (3,3)_0 & \Psi_Q^{(3)} &= (3,\bar{3})_{\frac{1}{3}} \\ D_{1,2}^{c(1,2)} &= (\bar{3},1)_{\frac{1}{3}} & B^{c(3)} &= (\bar{3},1)_{\frac{1}{3}} \\ U^{c(1,2)} &= (\bar{3},1)_{-\frac{2}{3}} & T_{1,2}^{c(3)} &= (\bar{3},1)_{-\frac{2}{3}} \end{aligned} \tag{3.10}$$

With this assignment of charges all anomalies vanish. To generate Yukawa couplings for the up-type quarks in the first two generations we couple them to $\bar{\chi}$ similarly to how we generated the bottom Yukawa. The down-type quarks have renormalizable Yukawa couplings with $\bar{\Phi}$

$$W_{Yukawa}^{(1,2)} = Y_d^{(1,2)} \bar{\Phi} \Psi_Q^{(1,2)} D^{c(1,2)} + \frac{Y_u^{(1,2)} \eta_1}{M} \Psi_Q^{(1,2)} (\Phi_1 \Phi_2) U^{c(1,2)} \tag{3.11}$$

Note that one of the down type Yukawa couplings has to be sizable in order to give large enough mass to the D' and S' partners. This coupling cannot be smaller than ~ 0.1 but it should not be too large either since these Yukawa couplings break the $\Phi \leftrightarrow \bar{\Phi}$ parity symmetry which is required to keep the Higgs naturally light. While there is room for both conditions to be met, we expect that the D' and S' partners are significantly lighter than $f \sim$ few TeV.

Note that embedding the first and second generation quarks differently into $\mathrm{SU}(3)_w$ representations also violates the approximate flavor symmetries of the MSSM and may lead to observable flavor changing effects.

3.3 Avoiding a Landau pole for the top Yukawa coupling

In the MSSM the electro-weak symmetry is broken by two multiplets H_u and H_d, with the relative size of the vacuum expectation values parameterized by $\tan\beta$. Since the top

Yukawa coupling $Y_T H_u Q_3 T^c$ involves only H_u, the coupling constant must be larger than the corresponding coupling in the standard model $Y_T = \lambda_t / \sin\beta$. The soft masses of H_u and H_d determine the relative size of the Higgs vevs so that for equal soft masses one finds $\tan\beta = 1$ and therefore $Y_T = \sqrt{2}\lambda_t$. This is a problem in the MSSM because the top Yukawa coupling reaches a Landau pole well before the unification scale.

The problem is even worse here. Even before introducing the fields Φ_3 and $\bar{\Phi}_3$, the Higgs vev is evenly distributed between Φ and $\bar{\Phi}$ which corresponds to $\tan\beta = 1$. However in order to generate a tree level quartic coupling for the Higgs our model requires additional mixing with the doublet in $\bar{\Phi}_3$ which further increases the required top Yukawa coupling. To explore the divergence of the Yukawa couplings quantitatively, we analyze the RGEs of Y_1 and Y_2. Ignoring the small $U(1)_X$ gauge coupling and all other Yukawa couplings they are

$$\begin{aligned}
\frac{d}{dt}Y_1 &= \frac{Y_1}{16\pi^2}\left[7Y_1^2 + Y_2^2 - \frac{16}{3}(g_s^2 + g_w^2)\right] \\
\frac{d}{dt}Y_2 &= \frac{Y_2}{16\pi^2}\left[7Y_2^2 + Y_1^2 - \frac{16}{3}(g_s^2 + g_w^2)\right] .
\end{aligned} \tag{3.12}$$

As in the MSSM, there is an infrared quasi fixed point of the top Yukawa couplings at $Y_i^2 \sim \frac{2}{3}(g_s^2 + g_w^2)$. Numerically, this fixed point is almost identical to the MSSM fixed point at $\frac{1}{18}(16g_s^2 + 9g_w^2)$. Using $Y_1 = \sqrt{2}y_t$ at the TeV scale, we find that Y_1 blows up near 10^7 GeV.

The root of the problem lies in the significant mixing between several Higgs doublets, so that the strength of the top Yukawa coupling is diluted, and larger values the primordial Yukawa couplings Y_i are required. To obtain these larger values without encountering Landau poles, we modify the renormalization group equations by splitting the color group $\mathrm{SU}(3)_c$ into two. In the UV we take the third generation to transform under $\mathrm{SU}(3)_{c1}$ whereas the two light generations transform under $\mathrm{SU}(3)_{c2}$. At energies of order 5-10 TeV the two groups are broken to the diagonal by expectation values for vector-like bi-fundamentals V and $\bar{V}$.

As can be seen from the matching relation

$$\frac{1}{g_s^2} = \frac{1}{g_{c1}^2} + \frac{1}{g_{c2}^2} \tag{3.13}$$

the coupling constant g_{c1} which now replaces g_s in the RGEs can be significantly larger. In particular, since the one-loop beta function for g_{c1}^2 vanishes exactly, g_{c1}^2 can even be a few times larger than g_s^2 and yet remain perturbative. Larger contributions from the gauge interactions in eq. (3.12) in turn predict larger values of the fixed point Yukawa coupling. In other words, much larger low energy values for Y_1 and Y_2 are now consistent with perturbativity all the way to the Planck scale.

Explicitly, the new (anomaly free) quantum numbers or quark and lepton superfields under $(\mathrm{SU}(3)_{c1}, \mathrm{SU}(3)_{c2}, \mathrm{SU}(3)_w)_{U(1)_X}$ are

$$\Psi_Q^{(1,2)} = (1,3,3)_0 \qquad \Psi_Q^{(3)} = (3,1,\bar{3})_{\frac{1}{3}} \qquad \Psi_Q^{(4)} = (3,1,3)_0$$

$$
\begin{aligned}
D^{c(1,2)}_{1,2} &= (1,\bar{3},1)_{\frac{1}{3}} \qquad & B^c &= (\bar{3},1,1)_{\frac{1}{3}} \qquad & B^c_2 &= (\bar{3},1,1)_{\frac{1}{3}} \\
U^{c(1,2)} &= (1,\bar{3},1)_{-\frac{2}{3}} \qquad & T^c_{1,2} &= (\bar{3},1,1)_{-\frac{2}{3}} \qquad & V &= (3,\bar{3},1)_0 \ .
\end{aligned} \tag{3.14}
$$

In order to generate mixing between the third generation quarks and those of the first and second generation (V_{ub}, V_{cb}, V_{td} and V_{ts}) additional mixing between the quarks with different color groups is needed. This problem is easily solved with the help of the extra colored multiplets which we introduced to implement the charge conjugation Z_2 symmetry in the third generation. They allow us to write renormalizable cross-generation Yukawa couplings. To be specific,

$$
W_{\Psi_{Q4}} = \xi_{1,2}\bar{\Psi}_{Q_4} V \Psi_{Q_{1,2}} + Y B^c \bar{\Phi}\Psi_{Q_4} + M_c \bar{\Psi}_{Q_4}\Psi_{Q_4} \tag{3.15}
$$

and below M_c, Ψ_{Q_4} and $\bar{\Psi}_{Q_4}$ can be integrated out to yield the desired off-diagonal Yukawa couplings

$$
W^{\text{eff}}_{\Psi_{Q4}} = -\left(\frac{f_c}{M_c}\xi_{1,2} Y\right) B^c \bar{\Phi}\Psi_{Q_{1,2}} \quad . \tag{3.16}
$$

4. Phenomenology and outlook

This model offers very rich phenomenology. In addition to superpartners we expect to see new gauge bosons and fermions from the little Higgs mechanism and new fields associated with the doubled color sector. There are four primary scales of phenomenology - the scale f_c where the two different color gauge groups are broken to the SM color group, the little Higgs scale f, the scale of soft scalar masses, and the gaugino/Higgsino masses. In figure (1) we have depicted one possible hierarchy.

At f_c we find extra color gauge bosons, the axi-gluons. Their mass squared is given by $\sim (g^2_{c1} + g^2_{c2}) f^2_c$ which is heavier than f^2_c because of the large values of the color gauge coupling constants. Axi-gluons mediate $B_0 - \bar{B}_0$ mixing at tree level which implies a lower bound on the scale f_c. Assuming that the new contribution to the $B_0 - \bar{B}_0$ mixing amplitude is less than or equal to the SM amplitude we find roughly $f_c \geq 5\,\text{TeV}$. The LHC will not be able to produce such heavy axi-gluons with significant cross section.

At the little Higgs scale f, we expect to find new charged gauge bosons (W's), neutral gauge bosons (Z') as well as new chiral multiplets which were needed in order to embed matter particles into full $\text{SU}(3)_w$ multiplets ($T - \bar{T}$ for example). A lower bound on the scale f can be obtained from contributions of the Z' to precision electroweak observables. Previous studies obtained bounds on the order of $f \gtrsim 3\,\text{TeV}$ which is the scale we have used for the explicit example in the previous section. Naturalness implies that the little Higgs scale is near this lower limit, and we expect that the LHC will be able to produce W's, Z' as well as little quarks. For details on the phenomenology of the Simplest Higgs see [21, 33−37].

Further lower in the energy scale we expect a range of superpartners along with an extra set of electroweak doublets from our non-minimal Higgs sector. As is seen in Fig(1) the scalar superpartners are expected to be heavier than the gauginos. LHC will certainly

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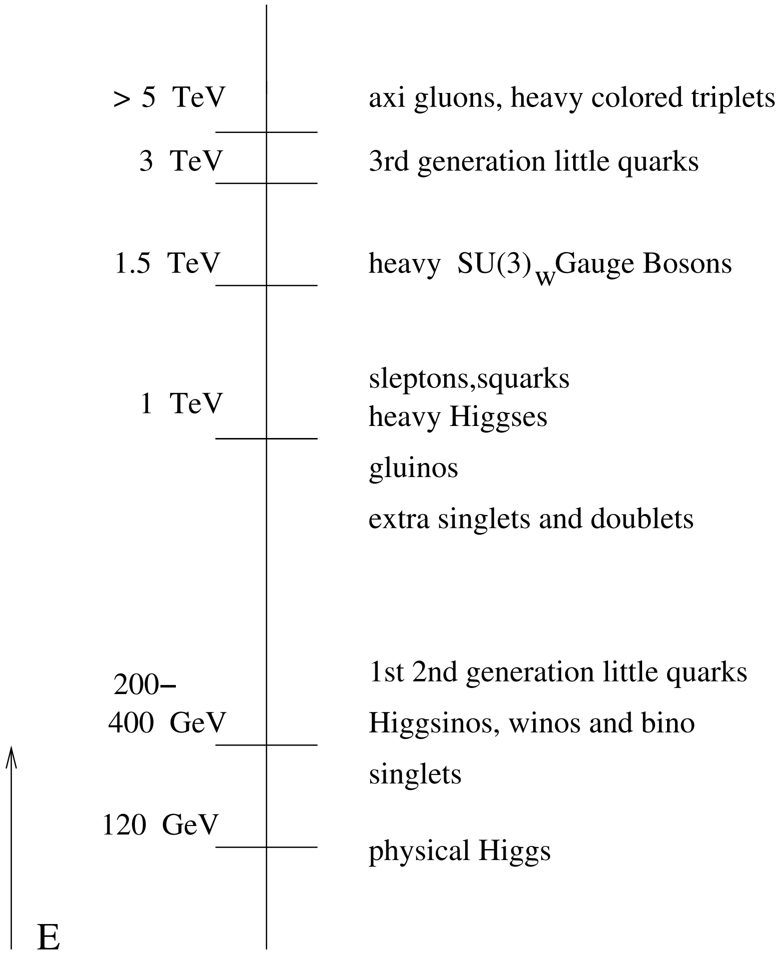

Figure 1: A possible spectrum of the theory

produce $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{g}$ and possibly even $\tilde{q}\tilde{q}$ pairs, which then decay via the usual susy cascades. Chargino and neutralino production through valence quark annihilation into weak bosons may also be significant. Since sleptons are even heavier than squarks, slepton pair production is rather small.

Note added

While this manuscript was being prepared for publication we received references [38, 39] in which similar models were constructed.

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